# 9.2

## Binomial Theorem

**The binomial coefficients** that appear in the expansion of $(a + b)^n$ are the values of ${}_nC_r$ for r = 0, 1, 2, 3, … n

A classical notation for ${}_nC_r$ especially in the context of binomial coefficients is $\binom{n}{r}$. Both notations are read 'n choose r".

**Using ${}_nC_r$ to Expand a Binomial:**

Example:

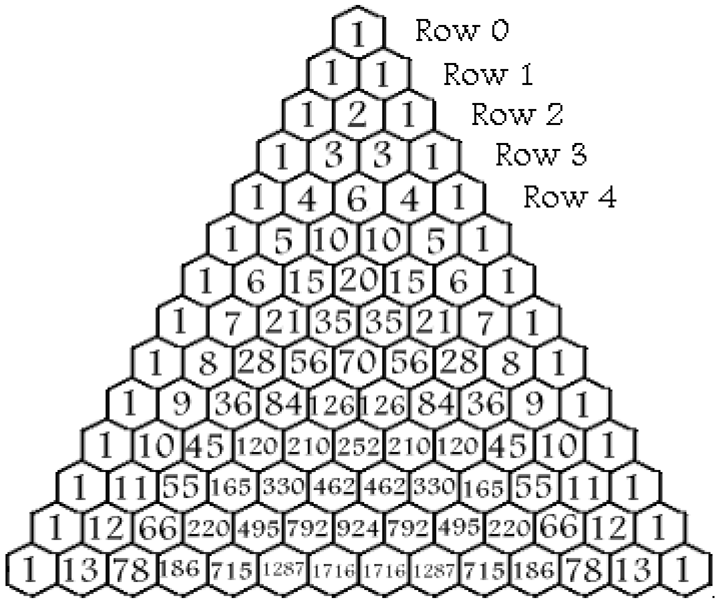
Expand $(a+b)^5$, using a calculator to compute the binomial coefficients.

**Solution:** Enter 5 ${}_nC_r$ {0, 1, 2, 3, 4, 5} into the calculator to find the binomial coefficients for n = 5. You should get the list {1, 5, 10, 10, 5, 1}.

Using these coefficients we get the expansion:

$$(a+b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$$

## Pascal's triangle




## The Binomial Theorem

For any positive integer n

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{n}b^n$$

$$where \binom{n}{r} = {}_nC_r = \frac{n!}{r!(n-r)!}$$

## Example: Expanding a Binomial

Expand $\left(2x - y^2\right)^4$ by hand using the Binomial Theorem.

Solution: a = 2x and b = $-y^2$.

Using: $\left(a+b\right)^4 = 1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$$\left(2x-y^2\right)^4=\left(2x\right)^4+4(2x)^3\left(-y^2\right)+6\left(2x\right)^2\left(-y^2\right)^2+4\left(2x\right)\left(-y^2\right)^3+\left(-y^2\right)^4$$

$$=16x^4-32x^3y^2+24x^2y^4-8xy^6+y^8$$

**Computing Binomial Coefficients:**

Find the coefficient of $x^{10}$ in the expansion of $(x + 2)^{15}$.

Solution: The only term in the expansion that we need to deal with is ${}_{15}C_{10}\,x^{10}2^5$.

$${}_{15}C_{10}\,x^{10}2^5 = 3003\cdot 32\cdot x^{10} = 96{,}096x^{10}.$$